VALUATION

Valuing Loan Guarantees

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Derivatives such as credit default swaps (CDSs) have recently been front-page news. Derivatives can be effective vehicles for hedging, but when used improperly can have disastrous results. Although the mathematics of derivative valuation can be very complex, using derivatives as an alternative valuation technique can be a valuable tool for valuators. In this article I will use a CDS as a tool to value a loan guarantee.

There are several reasons why an appraiser would want to value the guarantee, including: purchase price allocation, intercompany expense allocation, sale-related valuation adjustments, and allocating a guaranteed loan's principal between the portion that is debt and the portion that is equity.

The best way to understand the process is to work through an example.

ABC Company is a wholly owned subsidiary of XYZ Corporation. ABC borrows $300,000 from Regional Bank to purchase equipment. ABC's debt obligation to Regional Bank is guaranteed by XYZ, which is more creditworthy than ABC and has a Moody's BBB credit rating. Because of XYZ's creditworthiness, the interest rate that ABC pays is the BBB interest rate of 8 percent. If XYZ had an AAA credit rating (the highest Moody's rating where the historical default probability is near zero), the interest rate paid by ABC would have been the AAA rate of 6 percent. XYZ hires an appraiser to value the loan guarantee.

The terms of the loan agreement require ABC to make payments of $100,000 at the end of year one, $100,000 at the end of year two, and $153,274 at the end of year three. The loan amortization schedule applicable to ABC's debt obligation at the 8 percent contractual interest rate is shown in Table 1.

If ABC defaults on the loan, Regional Bank will take possession of the equipment, sell it at auction, and use the auction proceeds to reduce the balance of the loan. ABC estimates that the current auction value of the equipment is $250,000 and that it will depreciate at an annual rate of 30 percent. The estimated auction value of the equipment at each payment date is shown in Table 2.

The terms of the loan guarantee require XYZ to pay off ABC's outstanding loan balance to Regional Bank should

TABLE 1: LOAN AMORTIZATION SCHEDULE

Year	Loan Balance	Payment	Principal	Interest
0	300,000	--	--	--
1	224,000	100,000	76,000	24,000
2	141,920	100,000	82,080	17,920
3	0	153,274	141,920	11,354

TABLE 2: ESTIMATED AUCTION VALUE OF EQUIPMENT AT EACH PAYMENT DATE

Year	Auction Value	Depreciation
0	250,000	--
1	175,000	75,000
2	122,500	52,500
3	85,750	36,750

ABC default. The dollar amount of the obligation depends on when the default occurs. For example, if ABC defaults at the first payment date, the balance of the loan would be the original $300,000 borrowed plus $24,000 of accrued interest (see Table 1). Regional Bank would take possession of the equipment and sell it at auction for $175,000 (see Table 2). Regional bank would then apply the auction proceeds to reduce ABC's loan balance from $324,000 to $149,000. XYZ's obligation to Regional Bank under the terms of the loan guarantee is $149,000, or the balance of the loan. XYZ's estimated contingent loss by year of default is shown in Table 3.

TABLE 3: XYZ'S ESTIMATED CONTINGENT LOSS

YEAR	PRINCIPAL	INTEREST	AUCTION PROCEEDS	XYZ LOSS
1	300,000	24,000	175,000	149,000
2	224,000	17,920	122,500	119,420
3	141,920	11,354	85,750	67,524

According to Table 3, XYZ has a significant liability if ABC defaults. The value of the loan guarantee would be the compensation demanded by XYZ for taking this risk. If ABC had to go out and purchase the loan guarantee from XYZ, assuming perfect markets, the price of such a guarantee would equal the cost incurred by XYZ in hedging the risk associated with the guarantee.

The appraiser determines that the loan guarantee will be valued as a CDS, a bilateral contract in which one party (the protection buyer) pays a fee in return for a contingent payment by the other party (the protection seller) following a credit event of a reference security. The credit event in this case would be default, the reference security would be the Regional Bank loan, and the mechanism used to determine the payment is Table 3. The protection buyer is ABC and the protection seller is XYZ. The value of the CDS represents the amount that XYZ would need to fund the hedge such that once the hedge was funded there would be enough money in the hedge to pay off Regional Bank should ABC default, and zero in the hedge should ABC not default.

The appraiser's first attempt at valuation is the market approach, which tries to price the swap based on the prices at which CDSs currently trade. There are problems with this approach. The appraiser does not know where ABC fits on the Moody's ratings scale, which is critical because the swap premium increases as credit rating decreases. The appraiser must also factor in a loan amortization and recovery rate that is different from what is being assumed in the market quote. Because of these limitations, the appraiser concludes that the primary valuation method will be to value the swap directly.

VALUATION ASSUMPTIONS

One assumption we make is that of complete markets. A financial marketplace is said to be complete when a market exists with an equilibrium price for every asset in every possible state of the world. A complete market assumption allows the hedge to replicate the CDS by trading in the risk-free loan and the underlying risky loan. Note that if the appraiser is unwilling to make the complete market assumption in this case, then the appraiser cannot use the Black-Scholes option pricing model (or the lattice model, which is the discrete time equivalent), which makes the same assumption.

Another assumption we make is that the market rate of interest at which ABC could borrow absent the loan guarantee is known. For this exercise, I will assume that this rate cannot be ascertained directly (i.e., Regional Bank does not quote an interest rate with and without the guarantee). As we don't have a quoted rate, we will determine a proxy rate analytically. To do so, we first estimate the volatility of asset returns (asset return before financing costs such as interest expense). One way to estimate volatility is to determine the CAPM Equity Beta for a publicly traded comparable company, convert the CAPM Equity Beta into a Total Equity Beta, and then de-lever. For this exercise we will assume that the annual volatility of asset returns is 40 percent.

Other assumptions being made are that the risk-free lending rate is the AAA rate of 6 percent, there are no loan prepayments, and the loss-given-default (XYZ's liability to Regional Bank should ABC default) is according to Table 3.

VALUING DERIVATIVE ASSETS

Derivatives are valued via no-arbitrage pricing. Because derivative assets derive their value from the underlying asset, the derivative can be replicated by trading in the underlying risky asset and a risk-free asset. An arbitrage exists if a portfolio that consists of the derivative asset and the replicating portfolio can be set up at zero cost today, has a zero probability of loss in the future, and has a positive probability of gain in the future. If the derivative is too cheap, then the trading strategy to exploit the arbitrage would be to take a long position in the derivative asset and a short position in the replicating portfolio. Conversely, if the derivative is too expensive, then the trading strategy would be to take a short position in the derivative asset and a long position in the replicating portfolio. The goal of pricing derivatives is to derive the no-arbitrage price of the derivative asset directly.

Note: We will follow the same methodology that Black, Scholes, and Merton used to develop the Black-Scholes option pricing model. The mathematics presented below is for illustrative purposes only, not a complete derivation of the CDS pricing equation.

LEGEND OF SYMBOLS

- B_t = Value of the risk-free loan at time t
- C_t = Value of the CDS at time t
- C_n = Value of the CDS at payment date should ABC not default
- C_d = Value of the CDS at payment date should ABC default
- L_t = Value of the ABC loan at time t
- L_n = Value of the ABC loan at payment date should ABC not default
- L_d = Value of the ABC loan at payment date should ABC default
- r = Risk-free interest rate

The CDS Pricing Equation. A hedge portfolio comprised of a position in the derivative asset and a position in the underlying asset is created. The portfolio asset weights are set such that any change in the value of the derivative asset is offset by the same change in the value of the underlying asset, making the portfolio risk-free. A risk-free portfolio presents no opportunities for arbitrage. From this portfolio, a PDE (partial differential equation that describes how portfolio value changes) and a solution to the PDE are derived. The PDE and the solution for our hedge portfolio are as follows:

$$\text{PDE: } r\frac{\Delta C_t}{\Delta L_t}L_t + \frac{\Delta C_t}{\Delta t} - rC_t = 0;$$

$$\text{Solution: } C_t = \Theta_1 B_0(1+rt) - \Theta_2 L_t$$

The solution to the PDE says that a CDS can be replicated using a risk-free loan and the underlying risky loan. At loan origination and after each payment is received (except for the final payment), a hedge is created. At each payment date (before the payment is received or the loan defaults), the hedge is unwound. We know the value of the CDS at each payment date given default or no default (the boundary conditions). This gives us two equations and two unknowns (the thetas in the solution equation above). We can now solve for the portfolio asset weights which are:

$$\Theta_2 = \frac{C_d - C_n}{L_n - L_d} \quad ; \quad \Theta_1 = \frac{\Theta_2 L_d + C_d}{B_0(1+rt)}$$

We can now solve for the value of the CDS. The equation for the value of the CDS at time zero is:

$$C_0 = \Theta_1 B_0 - \Theta_2 L_0$$

In summary, to price the CDS when the hedge is set up, we need the portfolio asset weights and the market values of the risky and risk-free loans at that time.

RISKY AND RISK-FREE LOAN VALUES

Since the hedge portfolio will take positions in both the risk-free and risky loans, we need a market value schedule for both assets. The value of the risk-free loan is the contractual cash flow from the ABC loan discounted at the risk-free rate of 6 percent, or $312,031. Table 4 shows the loan amortization schedule applicable to the risk-free loan.

The credit spread is the interest rate that a lender charges to a borrower over and above the risk-free lending rate, and is a function of the probability of borrower default, the loss given borrower default and a premium for default risk. There are a number of models that we can use to quantify default risk. The simplest is the Black-Scholes-Merton Default Model, where the firm defaults when the market value of assets is below a given default point. That point generally lies somewhere between total liabilities and current liabilities. For this exercise we will define the default point to be the book value of total liabilities. The probability that the firm will default is a function of (a) the distance between asset value and the default point and (b) the volatility of assets. The market value

TABLE 4: LOAN AMORTIZATION SCHEDULE FOR RISK-FREE LOAN

Year	Loan Balance	Payment	Principal	Interest
0	312,031	--	--	--
1	230,753	100,000	81,278	18,722
2	144,598	100,000	86,155	13,845
3	0	153,274	144,598	8,676

TABLE 5: MARKET VALUE OF ASSETS AND BOOK VALUE OF LIABILITIES

	Pre-Transaction	Transaction	Post-Transaction
Market value of assets	1,700,000	300,000	2,000,000
Book value of liabilities	800,000	300,000	1,100,000

of assets and the book value of total liabilities before and after the transaction with Regional Bank are shown in Table 5.

We will model asset value as a geometric Brownian motion, which is how asset prices are modeled in the Black-Scholes option pricing model. The equation for asset value (*A*) at any time *t* is:

$$A_t = A_0 \times Exp((m - 0.5\sigma^2)t + \sigma\sqrt{t}z)$$

In the asset value equation above, *m* is the expected asset return, sigma is the volatility of asset returns, *t* is time in years, and *z* is a normally-distributed random variate with mean zero and variance one. The distance to default is the number of standard deviations (the variable *z*) such that asset value is less than the default point (*D*). If we use the asset value equation above, we will get the actual probability of default—but we still need a default risk premium. If we change the probability measure to the risk-neutral measure, then the probability of default will include a premium for default risk. We do this by substituting the risk-free rate *r* for the expected asset return m in the equation above. The one-year distance to default for ABC is (*Formula A*):

FORMULA A

$$z = \frac{Log(D/A_0) - (r - 0.5\sigma^2)t}{\sigma\sqrt{t}} = \frac{Log(1100/2000) - 0.06 + 0.5 \times 0.40^2}{0.40} = -1.44$$

FORMULA B

$$proxy = \frac{(1+r) - (1-LGD) \times P}{(1-P)} - 1 = \frac{(1+0.06) - (1-0.45) \times 0.0743}{(1-0.0743)} - 1 = 0.1009 \approx 10\%$$

TABLE 6: LOAN AMORTIZATION SCHEDULE FOR RISKY LOAN

Year	Loan Balance	Payment	Principal	Interest
0	288,710	--	--	--
1	217,581	100,000	71,129	28,871
2	139,340	100,000	78,241	21,759
3	0	153,274	139,340	13,934

The risk-neutral probability of default (*P*) is the cumulative standard normal distribution function of *z*, or

$$P = N[z] = N[-1.44] = 0.0743$$

For every dollar that Regional Bank lends to ABC it will receive either one dollar times one plus the lending rate, or one dollar times one minus the loss-given-default (LGD), which is approximately 45 percent. We will use the risk-neutral default probability calculated above and solve for the lending rate. The proxy for the Regional Bank lending rate absent the loan guarantee is shown in *Formula B*.

The value of the risky loan is the contractual cash flow from the ABC loan discounted at the pre-guarantee market proxy rate of 10 percent, or $288,710. The loan amortization schedule applicable to the risky loan is shown in Table 6.

HEDGE MECHANICS

At loan origination, XYZ makes a one-time payment into the hedge. This payment represents a short position in the CDS. The hedge uses this cash plus the cash from a short position in the ABC loan to fund a long position in the risk-free loan.

At each payment date just before payment is made, ABC can either make the scheduled payment or default. If ABC does not default, then the value of the short position in the ABC loan is the loan principal balance plus accrued interest, and the value of the short position in the CDS is zero. If ABC does default, then the value of the short position in the ABC loan is the expected recovery from selling the collateral, and the value of the short position in the CDS is the expected payment to Regional Bank in satisfaction of the loan guarantee. The hedge portfolio is unwound such that long positions receive cash and short positions disburse cash. The remaining cash balance in the portfolio is zero if ABC defaults, and is equal to the amount of cash needed to set up the hedge in the next period should ABC not default.

At each payment date (except for the final payment date), if ABC defaults then there are no further transactions; if ABC does not default, then the hedge is set up for the next period.

VALUING THE GUARANTEE

We will now value the loan guarantee as a CDS. To do this we start with the last payment and work our way backwards to the first payment. We work backwards because we need to know how much cash must be in the hedge portfolio at each period-end, so that the hedge portfolio can be funded in the next period should ABC not default (i.e., is self-financing). Please see Table 7.

The amount needed to fund the hedge is the value of the credit default swap at the beginning of year one, or $22,641.

Using year one as an example:

(1) $$\Theta_2 = \frac{C_d - C_n}{L_n - L_d} = \frac{149{,}000 - 12{,}983}{317{,}581 - 175{,}000} \approx 0.9540$$

(2) $$\Theta_1 = \frac{\Theta_2 L_d + C_d}{B_0(1+rt)} = \frac{(0.9540 \times 175{,}000) + 149{,}000}{312{,}031 \times 1.06} \approx 0.9552$$

(3) $$C_0 = \Theta_1 B_0 - \Theta_2 L_0 = (0.9552 \times 312{,}031) - (0.9540 \times 288{,}710) \approx 22{,}641$$

To illustrate how the hedge works, we will work through what the cash flows are when we set up the hedge at the beginning of year one, and when we unwind the hedge at the end of year one should the loan default or not default.

Set up the hedge at the beginning of year one, as shown in Table 8.

Unwind the hedge at the end of year one should ABC default, as in Table 9.

TABLE 8: SET UP HEDGE AT BEGINNING OF YEAR 1

Make initial cash deposit into the hedge	22,641
Sell risky loans short and receive cash (0.9540 x 288,710)	275,418
Buy risk-free loans and disburse cash (0.9552 x 312,031)	-298,059
Portfolio value beginning of year	0

TABLE 9: UNWIND HEDGE AT END OF YEAR 1 IF ABC DEFAULTS

Payoff ABC loan and disburse cash to Regional Bank	-149,000
Cover risky loans sold short and disburse cash (0.9540 x 175,000)	-166,943
Sell risk-free loans owned and receive cash (0.9552 x 312,031 x 1.06)	315,943
Portfolio value end of year	0

TABLE 7: VALUING THE CREDIT DEFAULT SWAP

Description	Symbol	Year 3	Year 2	Year 1
CDS payment – No default	C_n	0	5,258	12,983
CDS payment – Default	C_d	67,524	119,420	149,000
Risky loan value – No default	L_n	153,274	239,340	317,581
Risky loan value – Default	L_d	85,750	122,500	175,000
Risky loan value – Begin year	L_0	139,340	217,581	288,710
Risk-free loan value – Begin year	B_0	144,598	230,753	312,031
Risk-free loan equation weight (2)	Θ_1	1.0000	0.9776	0.9552
Risky loan equation weight (1)	Θ_2	1.0000	0.9771	0.9540
CDS value – Begin year (3)	C_0	5,258	12,983	22,641

TABLE 10:
UNWIND HEDGE AT END OF YEAR 1 IF ABC DOES NOT DEFAULT

Cover risky loans sold short and disburse cash (0.9540 x 317,581)	-302,960
Sell risk-free loans owned and receive cash (0.9552 x 312,031 x 1.06)	315,943
Portfolio value end of year—rolled over to fund hedge in year 2	12,983

Unwind the hedge at the end of year one should ABC not default (Table 10).

CONCLUSION

The value of the loan guarantee evaluated as a CDS is $22,641 (see Table 7). If XYZ deposits this amount into a hedge, then the cash required of XYZ in the future should ABC default is zero.

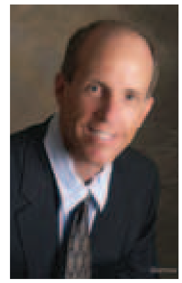
ABC may also consider recording the debt on its books as follow:

Portion that is equity	22,641
Portion that is debt	277,359
Total	300,000

Because the hedge is taking a short position in the risky loan and a long position in the risk-free loan, the value of the CDS should approximate the difference between the present values of the contractual loan payments at the market risk-free and risk rates, or

PV of pmts at 6% - PV of pmts at 10% = $312,031 – $288,710 = $23,321

This approximation is less and less accurate as the number of payments and the probability of default increases. For example, if we convert the Regional Bank loan to a one with 36 monthly payments and use a market interest rate of 20 percent (i.e., the loan is riskier), the value of the CDS is $64,820 and the difference between the present values of the contractual loan payments at the market risk-free and risk rates is $77,057. VE



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